Propositional Logic Solutions

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November 18, 2023

1. Given n propositional statements, how many possible combinations of their states exist? In the case we have exactly two propositions P and Q ($n = 2$), what are the possible combinations?

2. Convert the following sentences to expressions in propositional logic. You should need at most three propositions per statement. Please specify what the propositions are.

   (a) It rains.

   (b) It rains or it doesn't rain.

   (c) If martians exist, they have red hair.

   (d) He has an Ace only if he does not have a Knight or a Spade

   (e) New allocation not being worse than initial endowment is a necessary and sufficient condition for Pareto efficiency.

   (f) If independence implies uncorrelatedness, correlatedness implies dependence.

3. Which of the following are valid deductions?

   (a) Assertion: If I leave my umbrella at home, I will get wet today.
   Hypothesis 1: I got wet, so I must have left my umbrella at home.

Hypothesis 2: I didn't leave my umbrella at home, so I will not get wet today.
Hypothesis 3: I did not get wet today, so I must not have left my umbrella at home.

(b) Assertion: If two random variables X and Y are independent, their correlation is zero.
Hypothesis 1: X and Y have zero correlation, so they are independent.
Hypothesis 2: X and Y don't have zero correlation, so they must not be dependent.
Hypothesis 3: X and Y are dependent, so their correlation is not zero.

4. Simplify the following Boolean expression such that it only has three boolean operators:
$$\neg(A \vee \neg C) \vee ((\neg A \wedge B) \vee B)$$

5. Show that $P \implies Q$ is an equivalent statement to $\neg P \vee Q$ by filling in the following truth table:

6. Fill in the following truth table. Which binary operation have you derived in the last column? (From $\vee, \wedge, \implies, \iff$)

7. What is the boolean expression equivalent to "MUX Out" below? Draw the circuit that corresponds with this expression. The circuit is known as a multiplexer.

P	Q	S	MUX Out
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	0

8. Prove that $(P \Rightarrow Q) \Rightarrow ((Q \Rightarrow \perp) \Rightarrow (P \Rightarrow \perp))$. The consequent is known as the contrapositive.